

Sl. No. of Ques. Paper : 2320

GC-3

Unique Paper Code : 62354343

Name of Paper : Analytical Geometry and Applied Algebra

Name of Course : B.A. (Prog.) Mathematics (CBCS)

Semester : III

Duration : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.
Attempt any two parts from each question.

SET-C

1. (a) Identify and sketch the curve:

$$x = y^2 - 4y + 2$$

and also label the focus, vertex and directrix.

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- (b) Describe the graph of the curve:

$$3(x+2)^2 + 4(y+1)^2 = 12$$

Also find its centre and foci.

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- (c) Describe the graph of the hyperbola:

$$x^2 - y^2 - 4x + 8y - 21 = 0$$

And sketch its graph.

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2. (a) Find the equation of the parabola that has its vertex at (1,2) and focus at (4,2). Also

state the reflection property of parabola.

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- (b) Find the equation of the ellipse whose length of major axis is 26 and foci ($\pm 5, 0$) and

also sketch it.

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- (c) Find and sketch the curve of the hyperbola whose foci are (6,4) and (-4,4) and

eccentricity is 2.

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3. (a) Consider the equation:

$$3x^2 + 2xy + 3y^2 = 19.$$

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Rotate the coordinate axes to remove the xy -term. Then identify the type of conic represented by the equation and sketch its graph.

(b) Let an $x'y'$ - coordinate system be obtained by rotating an xy - coordinate system through an angle $\theta = 30^\circ$.

(i) Find the $x'y'$ - coordinate of the point whose xy - coordinates are (2, 4).

(ii) Find an equation of the curve $2x^2 + 2\sqrt{3}xy = 3$ in $x'y'$ - coordinates.

(c) Find the equation of two spheres that are centered at the origin and are tangent to the sphere of radius 1 centered at $(0, 0, 7)$.

4(a) (i) Find a vector of length 9 and oppositely directed to $\mathbf{v} = -5\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$.

(ii) Sketch the surface $2x + z = 3$ in 3-space.

(b) (i) Find the vector component of $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ orthogonal to $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 28\mathbf{k}$.

(ii) Find the area of triangle with vertices $P(2, 0, -3)$, $Q(1, 4, 5)$, $R(7, 2, 9)$.

(c) Prove that

$$\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$

and interpret the result geometrically.

5 (a) Let L_1 and L_2 be the lines whose parametric equations are

$$L_1 : x = 4t \quad y = 1 - 2t \quad z = 2 + 2t$$

$$L_2 : x = 1 + t \quad y = 1 - t \quad z = -1 + 4t$$

(i) Show that the lines L_1 and L_2 intersect at the point $(2, 0, 3)$.

(ii) Find the parametric equation of line that is perpendicular to L_1 and L_2 and passes through their point of intersection.

(b) (i) Determine whether the points $P_1(6, 9, 7)$, $P_2(9, 2, 0)$ and $P_3(0, -5, -3)$ lie on the same line.

(ii) Where does the line

$$x = 2 - t, \quad y = 3t, \quad z = -1 + 2t$$

intersect the plane $2y + 3z = 6$.

(c) (i) Find the equation of the plane through $(1, 4, 3)$ that is perpendicular to the line

$$x = 2 + t, \quad y + 3 = 2t, \quad z = -t.$$

(ii) Determine whether the planes

$$3x - 2y + z = 1, 4x + y - 2z = 4$$

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are parallel, perpendicular or neither.

$$3 + 3\frac{1}{2}$$

6. (a) Given three containers 3, 7, and 10 liters respectively with the largest being full of water, determine a minimum sequence of pouring method of dividing this quantity of water into two equal amounts of 5 liters using the three containers and no other measuring devices.

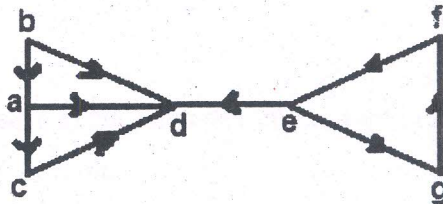
$$6\frac{1}{2}$$

- (b) Is the following square a Latin square? Can it be a group with the multiplication operation defined?

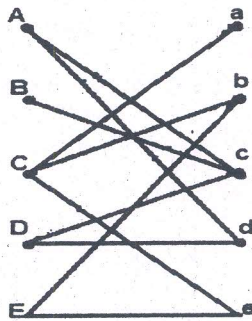
*	1	2	3	4	5
1	1	2	3	4	5
2	2	1	4	5	3
3	3	4	5	2	1
4	4	5	1	3	2
5	5	3	2	1	4

$$6\frac{1}{2}$$

- (c) (i) Given the influence model. Find the sets of minimum number of vertices which can influence every other vertex in the graph.



- (ii) Find a matching or explain why none exists for the following graph.



$$3 + 3\frac{1}{2}$$

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