[This question paper contains 10+6 printed pages.]

**Sr. No. of Question Paper : 7563** **F-7** **Your Roll No................**

Unique Paper Code : 2271303

Name of the Paper : Statistical Methods in Economics - II

Name of the Course : **B.A. (Hons.) Economics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions within each section are to be answered in a continuous manner on the answer sheet. Start each question on a new page and all subparts of a question should follow one after the other.
3. Use of simple calculator is permitted.
4. Required statistical tables are attached with this question paper.
5. This paper contains four sections. Attempt all sections.
6. Answers may be written either in English or Hindi; but the same medium should be used throughout the paper.

**छात्रों के लिए निर्देश**

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए ।
2. प्रत्येक भाग के सभी प्रश्न के उत्तर एक साथ उत्तर पुस्तिका पर दें। प्रत्येक प्रश्न नये पेज पर और उपभागों का एक के बाद क्रम से प्रश्नों के उत्तर दीजिए।
3. साधारण कैलकुलेटर का उपयोग मान्य है।
4. इस प्रश्न पत्र के साथ स्टेटिकल टेबल संलग्न की गई है।
5. इस प्रश्न पत्र में चार खंड हैं। सभी खंडों के उत्तर दीजिए।
6. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए ।

**Section I**

**Q1 is compulsory. Attempt any one From Q2 and Q3.**

**भाग I**

**प्रश्न संख्या 1 अनिवार्य है।**

**किन्हीं एक प्रश्न संख्या 2 और 3 को हल कीजिए।**

(a) A confectionary shop sells three types of candies priced at Rs 3.00, Rs 3.20, and Rs 3.40 per piece, respectively. Let $X_1, X_2$ and $X_3$ denote the number of these candies sold on a particular day. Suppose the $X_i$'s are independent with $\mu_1, \mu_2$ and $\mu_3$ equal to 1000, 500 and 300, respectively and $\sigma_1, \sigma_2$ and $\sigma_3$ equal to 100, 80 and 50, respectively.

(i) What is the expected daily revenue of the shop from the sale of these candies?

(ii) Calculate the standard deviation of the revenue from the sale of these candies.

(iii) Would your answers be correct if the $X_i$'s were not independent? Explain. (4)

(b) For the following sample data on variables x and y:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 112.3 | 97.0 | 92.7 | 86.0 | 102.0 | 99.2 | 95.8 | 103.5 | 89.0 | 86.7 |
| Y | 75.0 | 71.0 | 57.7 | 48.7 | 74.3 | 73.3 | 68.0 | 59.3 | 57.8 | 48.5 |

(i) Obtain the equation of the least squares line and interpret its slope.

(ii) Calculate and interpret the coefficient of determination. (6)

(क) एक मिठाई की दुकान तीन प्रकार की मिठाइयाँ बेचती है जिनका मूल्य क्रमशः 3.00 रु., 3.20 रु, एवं 3.40 रु. प्रति नग है। माना कि $X_1, X_2$ और $X_3$ किसी विशिष्ट, दिन बेची गयी मिठाइयों की संख्या को व्यक्त करते हैं। माना कि $X_i$ के मान $\mu_1, \mu_2$ और $\mu_3$ से स्वतंत्र हैं एवं क्रमशः 1000, 500 एवं 300 के बराबर हैं एवं $\sigma_1, \sigma_2$ और $\sigma_3$ क्रमशः 100, 80 एवं 50 के बराबर हैं।

(i) इन मिठाइयों की बिक्री से दुकान की अनुमानित दैनिक आय क्या है?

(ii) इन मिठाइयों की बिक्री से होने वाली आय के मानक विचलन का परिकलन कीजिए।

(iii) यदि $X_i$ के मान स्वतंत्र नहीं होते तो क्या आपके उत्तर सही होते? व्याख्या कीजिए।

(ग) x एवं y के चरों हेतु निम्नलिखित प्रतिदर्श आंकड़ों के लिए:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X | 112.3 | 97.0 | 92.7 | 86.0 | 102.0 | 99.2 | 95.8 | 103.5 | 89.0 | 86.7 |
| Y | 75.0 | 71.0 | 57.7 | 48.7 | 74.3 | 73.3 | 68.0 | 59.3 | 57.8 | 48.5 |

(i) least squares line का समीकरण ज्ञात कीजिए एवं उसकी प्रवणता की व्याख्या कीजिए।

(ii) निर्धारण गुणांक का परिकलन कीजिए एवं उसकी व्याख्या कीजिए।

2. (a) The time take by a randomly selected student to fill a form has a normal distribution with mean value 10 min and standard deviation 2 min. If five students fill a form on one day and six on another day, what is the probability that the sample average amount of time taken on each day is at most 11min? (5)

(b) For the following summary statistics:

$n=15,\ \sum x_i = 1640.1,\ \sum y_i = 299.8,\ \sum x_i^2 = 179{,}849.73$

$\sum y_i^2 = 6430.06,\ \sum x_i y_i = 32{,}308.59$

(i) Obtain the equation of the estimated regression line of y on x.

(ii) Use the estimated line used to predict y when x is 135.

(iii) Calculate and interpret a point estimate of σ.

(iv) What are the values of SSE and SST?

(v) What proportion of observed variation in y can be attributed to the approximate linear relationship between x and y? (10)

(क) यादृच्छिक रूप से चयनित किए गए छात्रों द्वारा एक प्रपत्र को भरने में लिया गया समय 10 मिनट के माध्य मान एवं 2 मिनट के मानक विचलन के साथ प्रसामान्य बंटन को प्रदर्शित करता है। यदि एक दिन पांच छात्र प्रपत्र भरते हैं एवं दूसरे दिन छः छात्र प्रपत्र भरते हैं तो, प्रपत्र भरन में प्रतिदिन लिए गए समय की प्रतिदर्श औसत मात्र अधिकतम 11 मिनट होने की प्रायिकता क्या है?

(ख) निम्नलिखित सारांशीकृत आँकड़ों के लिए:

$n=15,\ \sum x_i=1640.1,\ \sum y_i=299.8,\ \sum x_i^2=179{,}849.73$

$\sum y_i^2=6430.06,\ \sum x_i y_i=32{,}308.59$

(i) x पर y की अनुमानित समाश्रयण रेखा का समीकरण प्राप्त कीजिए।

(ii) y का अनुमान लगाने के लिए अनुमानित रेखा का उपयोग कीजिए जब कि x का मान 135 है।

(iii) σ के बिंदु आकल का परिकलन कीजिए एवं व्याख्या कीजिए।

(iv) SSE एवं SST के मान क्या हैं?

(v) y में प्रेक्षित विचरण के किस अनुपात को x एवं y के बीच अनुमानित रैखिक संबंध के लिए जिम्मेदार ठहराया जा सकता है?

3. (a) A College has three administrative departments each having two employees. Information regarding their monthly salaries (thousands of Rs) is as follows:

| Department | 1 | 1 | 2 | 2 | 3 | 3 |
|---|---|---|---|---|---|---|
| Employee | 1 | 2 | 3 | 4 | 5 | 6 |
| Salary | 30 | 34 | 28 | 32 | 42 | 22 |

Suppose one of the three departments is randomly selected. Let $X_1$ and $X_2$ denote the salaries of the two employees. Determine the sampling distribution of $\bar{X}$. Where is this distribution centered? (5)

(b) Following summary statistics are given to explain the relationship between y and x:

$$\sum x_i = 659, \sum x_i^2 = 28{,}967.50, \bar{x} = 36.6111, S_{xx} = 4840.7778,$$

$$\sum y_i = 293.2, \quad \sum x_i y_i = 9293.95, \quad \sum y_i^2 = 5335.76,$$
$$\widehat{\beta_1} = -0.2976, \widehat{\beta_0} = 27.183, \quad SSE = 131.2402,$$
$$r^2 = 0.766, \ s = 2.8640$$

Calculate a prediction interval for y with a prediction level of 95% when x is 45. (5)

(c) Show that the "point of averages" $(\bar{x}, \bar{y})$ lies on the estimated regression line. (5)

(क) एक कॉलेज में तीन प्रशासनिक विभाग हैं जिनमें से प्रत्येक में दो कर्मचारी हैं। उनके मासिक वेतनों (हजार रुपयों में) के संबंध में जानकारी इस प्रकार है:

| | | | | | | |
|---|---|---|---|---|---|---|
| विभाग | 1 | 1 | 2 | 2 | 3 | 3 |
| कर्मचारी | 1 | 2 | 3 | 4 | 5 | 6 |
| वेतन | 30 | 34 | 28 | 32 | 42 | 22 |

माना कि तीन में से एक विभाग का याण्च्छिक रूप से चयन किया जाता है। माना कि $X_1$ और $X_2$ दो कर्मचारियों के वेतन को व्यद्र करते हैं। $\bar{X}$ के प्रतिचयन बंटन को निर्धारित कीजिए। यह बंटन कहाँ केन्द्रित है?

(ख) y एवं x के बीच संबंध की व्याख्या करने के लिए निम्नलिखित सारांश आंकड़े दिए गए हैं:

$$\sum x_i = 659, \sum x_i^2 = 28{,}967.50, \bar{x} = 36.6111, S_{xx} = 4840.7778,$$

$$\sum y_i = 293.2, \quad \sum x_i y_i = 9293.95, \quad \sum y_i^2 = 5335.76,$$
$$\widehat{\beta_1} = -0.2976, \widehat{\beta_0} = 27.183, \quad SSE = 131.2402,$$
$$r^2 = 0.766, \ s = 2.8640$$

95% के प्रागुक्ति स्तर के साथ ल के लिए प्रागुक्ति अंतराल का परिकलन कीजिए जबकि x का मान 45 है।

(ग) प्रदर्शित कीजिए कि "औसत बिंदु (point of averages) $(\bar{x}, \bar{y})$ अनुमानित समाश्रयण रेखा (regression line) पर अवस्थित हैं।

## SECTION II

## ATTEMPT ALL QUESTIONS

## भाग II

## सभी प्रश्नों को हल कीजिए

4. (a) Explain properties of a good estimator. Consider a random sample $(X_1, X_2, X_3 \ldots X_n)$ from a population from a probability distribution function $f(x;\ \theta)$ If Expected value of an estimator $\hat{\theta}$ is equal to $(n/n+1)^*\theta$, what is the bias of this estimator? Find an estimator that is unbiased. (3+2)

(b) Consider a random sample $(X_1, X_2, X_3 \ldots X_n)$ from a population from a probability distribution function

$f(x;\ \theta) = 0.5(1 + x^*\theta)$. Where $-1 \le x \le 1$.

(i) Show that $\hat{\theta}$ = 3*sample mean is an unbiased estimator for σ.

(ii) If a sample size is 3 and the sample is (-1, 0, 2) give point estimate for $\theta$. (4+1)

(c) Let $(x_1, x_2 \ldots\ldots x_n)$ be a random sample from a population with mean μ and standard deviation σ. Show that $S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$ is unbiased estimator of population variance, denoted by $\sigma^2$.

(5)

(d) From a survey of 500 families, 340 were reported to like Star Plus channel.

Find a 90% confidence interval for the proportion of families that like Star Plus channel. What is the required sample size if we want to be 95% confident that our estimate of the proportion of families that like Star plus channel is within 0.02 of the true proportion? (5)

(e) Let X be a continuous random variable that is uniformly distributed over (0, A). Find the moment estimator of A. (3+2)

(क) अच्छे प्राक्कलक के गुणों की व्याख्या कीजिए। प्रायिकता वितरण फलन $(x;\ \theta)$ से एक समष्टि के याणच्छिक प्रतिदर्श $(X_1, X_2, X_3 \ldots X_n)$ पर विचार कीजिए। यदि प्राक्कलक $\hat{\theta}$ का अपेक्षित मान $(n/n+1)*\theta$ के समतुल्य है, तो इस प्राक्कलक की अभिनति क्या है? ऐसा प्राक्कलक ज्ञात कीजिए जो अनभिनत है।

(ख) एक समष्टि के यादृच्छिक प्रतिदर्श $(X_1, X_2, X_3 \ldots X_n)$ पर विचार कीजिए जिसका प्रायिकता वितरण फलन इस प्रकार है:

$f(x;\ \theta) = 0.5(1 + x*\theta)$. जहाँ $-1 \le x \le 1$.

(i) प्रदर्शित कीजिए कि $\hat{\theta}=3*$ प्रतिदर्श माध्य $\theta$ के लिए अनभिनत प्राक्कलक है।

(ii) यदि प्रतिदर्श आकार 3 है एवं प्रतिदर्श (-1, 0, 2) है, तो $\theta$ के लिए बिंदु आकल प्रदान कीजिए।

(ग) माना कि माध्य – एवं मानक विचलन $\sigma$ की समष्टि से लिया गया एक यादृच्छिक प्रतिदर्श $(x_1, x_2 \ldots\ldots x_n)$ है। प्रदर्शित कीजिए कि $S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$ ऐसे समष्टि प्रसरण (population variance) का अनभिनत प्राक्कलक है जिसे $\sigma^2$ से व्यक्त किया जाता है।

(घ) 500 परिवारों के सर्वेक्षण से यह रिपोर्ट प्राप्त हुई कि 340 परिवार स्टार प्लस को पसंद करते थे। स्टार प्लस चैनल को पसंद करने वाले परिवारों के लिए 90% विश्वास्यता अंतराल को ज्ञात कीजिए। यदि हम 95% सुनिश्चित होना चाहते हैं कि स्टारं प्लस चैनल को पसंद करने वाले परिवारों का हमारा अनुमान वास्तविक अनुपात के 0.02 के अंतर्गत है तो वांछित प्रतिदर्श आकार क्या है?

(ङ) माना कि X संतत याणक्षिक चर है जो (0, A) पर समान रूप से वितरित है। । का आघूर्ण प्राक्कलक (moment estimator) ज्ञात कीजिए।

## SECTION III

### Q5 IS COMPULSORY. ATTEMPT ANY TWO QUESTIONS OUT OF Q6, Q7 AND Q8.

### भाग III

### प्रश्न 5 अनिवार्य है। प्र.6, प्र.7 एवं प्र.8 में से कोई दो प्रश्न हल कीजिए।

5. What are the two types of errors in hypothesis testing? For a given sample size, can both errors be simultaneously reduced? Explain with the help of diagram. (5)

परिकल्पना परीक्षण (hypothesis testing) में दो प्रकार की त्रुटियाँ कौन सी हैं? क्या दिए गए प्रतिदर्श आकार के लिए दोनों त्रुटियों को एक साथ कम किया जा सकता है? आरेख की सहायता से समझाइए।

6. (a) A professor believes that a standard deviation of about 13 points on a hundred point exam indicates that the exam does a good job. He gave an exam to his class of 31 students. The mean score was 72.7 and standard deviation was 15.9. Does this exam meet the goodness criterion? Use $\alpha = 0.10$.

(b) For each of the following pairs of hypothesis indicate if the rules of setting up hypothesis are followed. If not, give reasons.

(i) Ho: $\mu = 100$ H1: $\mu \leq 100$

(ii) Ho: $\mu \neq 21$ H1: $\mu < 21$

(iii) Ho: $\mu = 12.5$ H1: $\mu < 12.8$

(iv) Ho: $\mu > 100$ H1: $\mu \neq 100$

(v) Ho: $\mu > 100$ H1: $\mu = 100$ (5+5)

(क) एक प्रोफेसर का विश्वास है कि 100 अंकों की परीक्षा में लगभग 13 अंकों का मानक विचलन यह इंगित करता है कि परीक्षा अच्छा कार्य करती है। उसने 31 छात्रों की अपनी कक्षा की एक परीक्षा ली। माध्य प्राप्तांक 72.7 थे एवं मानक विचलन 15.9 था। क्या यह परीक्षा अच्छाई के मानदण्ड (goodness criterion) को पूरा करती है। $\alpha = 0.10$ का प्रयोग कीजिए।

(ख) निम्नलिखित में से परिकल्पना के प्रत्येक युग्म के लिए इंगित कीजिए कि क्या परिकल्पना की स्थापना के नियमों का पालन किया जाता है। यदि नहीं तो कारण बताइए।

(i) Ho: $\mu = 100$ H1: $\mu \leq 100$

(ii) Ho: $\mu \neq 21$ H1: $\mu < 21$

(iii) Ho: $\mu = 12.5$ H1: $\mu < 12.8$

(iv) Ho: $\mu > 100$ H1: $\mu \neq 100$

(v) Ho: $\mu > 100$ H1: $\mu = 100$

7. (a) A quality inspector picks up 100 masks from the market to note that 14 do not work as they are unable to filter out air impurities. The manufacturer claims that only 10% of masks are unable to filter out air impurities. Using a significance level of 98% can the manufacturer's claim be supported? What is the p value of your test?

(b) A consultant needs to compare two populations, but he needs to know if the variances are same for them before he proceeds. He collects samples of size 10 from both populations to get standard deviations of 12.2 and 15.4. Using a 95% confidence level test for equality of variances in both populations. (5+5)

(क) एक गुणवत्ता निरीक्षक बाजार से 100 मास्क उठाता है और उसे ज्ञात होता है कि उनमें से 14 मास्क कार्य नहीं कर रहे हैं क्योंकि वे वायु की अशुद्धियों का निस्पंदन करने में असमर्थ हैं। विनिर्माता का दावा है कि केवल 10% मास्क ही वायु की अशुद्धियों का निस्पंदन करने में असमर्थ हैं। 98% के सार्थकता स्तर का उपयोग कर क्या विनिर्माता के दावे का समर्थन किया जा सकता है? आपके परीक्षण का p मान क्या है?

(ख) एक सलाहकार को दो समष्टियों की तुलना करने की आवश्यकता है, किन्तुं यह कार्य आरम्भ करने से पहले वह ज्ञात करना चाहता है कि क्या उनके लिए प्रसरण समान हैं। वह 12.2 और 15.4 का मानक विचलन प्राप्त करने के लिए दोनों समष्टियों से 10 के आकार के प्रतिदर्श एकत्रित करता है। 95% विश्वास स्तर का प्रयोग कर दोनों समष्टियों में प्रसरणों की समानता का परीक्षण कीजिए।

8. (a) A consultancy firm wants to check if average wages across males and females are different. They took samples that reveal the following information:

| Males | Females |
|---|---|
| Mean = 62.5 | Mean = 39.7 |
| Standard deviation = 23.7 | Standard deviation = 8.9 |
| Size = 175 | Size = 168 |

(i) Using a 90% confidence level can we argue that there is no difference in wages across males and females? Assume equal population variances.

(ii) Can we argue that males earn more than females, using a 98% confidence level?

(b) The CEO and HR head of a company argue over the no of yearly holidays taken by employees. The HR head argues that on average employees take more than 40 holidays in a year. To test this claim a sample of 15 employees is taken and sample average is found to be 41.17 with standard deviation is 4.71. Test the HR head's claim that employees take more than 40 yearly holidays ata 0.05 level of significance. (5+5)

(क) एक कंसल्टेंसी फर्म यह जाँच करना चाहती है कि क्या पुरुषों और महिलाओं के बीच औसत पारिश्रमिक भिन्न हैं। वे प्रतिदर्श ग्रहण करते हैं जो निम्नलिखित जानकारी प्रकट करते हैं:

| पुरुष | महिला |
|---|---|
| माध्य = 62.5 | माध्य = 39.7 |
| मानक विचलन = 23.7 | मानक विचलन = 8.9 |
| आकार = 175 | आकार = 168 |

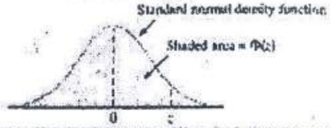
(i) 90% विश्वास स्तर का प्रयोग कर क्या हम यह तर्क कर सकते हैं कि पुरुषों एवं महिलाओं के पारिश्रमिक में कोई अंतर नहीं है। समष्टि प्रसरणों को समतुल्य मान लीजिए।

(ii) 98% विश्वास स्तर का प्रयोग कर क्या हम यह तर्क कर सकते हैं कि पुरुष महिलाओं की तुलना में अधिक अर्जित करते हैं?

(ख) कम्पनी के मुख्य कार्यकारी अधिकारी (CEO) एवं कम्पानी के एच.आर. प्रमुख कर्मचारियों द्वारा लिए गए वार्षिक अवकाशों के संबंध में तर्क करते हैं। एच.आर. प्रमुख का तर्क है कि औसत रूप से कर्मचारी वर्ष में 40 से अधिक अवकाश लेते हैं। इस दावे का परीक्षण करने के लिए 15 कर्मचारियों का एक प्रतिदर्श लिया जाता है एवं प्रतिदर्श का औसत 4.71 के मानक विचलन के साथ 41.17 पाया जाता है। एच.आर. प्रमुख के दावे का परीक्षण कीजिए कि कर्मचारी 0.05 के सार्थकता स्तर पर 40 से अधिक अवकाश लेते हैं।

**Table A.3 Standard Normal Curve Areas**

$\Phi(z) = P(Z \le z)$




| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0017 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0352 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3482 |
| −0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| −0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

*(continued)*

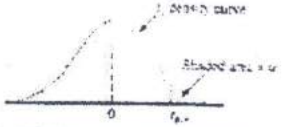


**Table A.3 Standard Normal Curve Areas *(cont.)***

$\Phi(z) = P(Z \le z)$

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9278 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

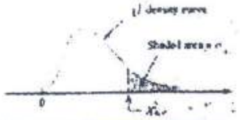
**Table A.5** Critical Values for t Distributions



| $\nu$ \ $\alpha$ | .10 | .05 | .025 | .01 | .005 | .001 | .0005 |
|---|---|---|---|---|---|---|---|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.31 | 636.62 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.326 | 31.598 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.213 | 12.924 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.767 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| 32 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 |
| 34 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 |
| 36 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 |
| 38 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.262 | 3.496 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |



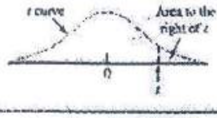
**Table A.7** Critical Values for Chi-Squared Distributions



| $\nu$ \ $\alpha$ | .995 | .99 | .975 | .95 | .90 | .10 | .05 | .025 | .01 | .005 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 2.706 | 3.843 | 5.025 | 6.637 | 7.882 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.992 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.344 | 12.837 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.832 | 15.085 | 16.748 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.440 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.012 | 18.474 | 20.276 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.534 | 20.090 | 21.954 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.022 | 21.665 | 23.587 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.724 | 26.755 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.041 | 19.812 | 22.362 | 24.735 | 27.687 | 29.817 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.600 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.577 | 32.799 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.407 | 7.564 | 8.682 | 10.085 | 24.769 | 27.587 | 30.190 | 33.408 | 35.716 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.843 | 7.632 | 8.906 | 10.117 | 11.651 | 27.203 | 30.143 | 32.852 | 36.190 | 38.580 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.033 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.670 | 35.478 | 38.930 | 41.399 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.042 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.195 | 11.688 | 13.090 | 14.848 | 32.007 | 35.172 | 38.075 | 41.637 | 44.179 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.558 |
| 25 | 10.519 | 11.523 | 13.120 | 14.611 | 16.473 | 34.381 | 37.652 | 40.646 | 44.313 | 46.925 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.807 | 12.878 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.194 | 46.962 | 49.642 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.120 | 14.256 | 16.147 | 17.708 | 19.768 | 39.087 | 42.557 | 45.772 | 49.586 | 52.333 |
| 30 | 13.787 | 14.954 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 31 | 14.457 | 15.655 | 17.538 | 19.280 | 21.433 | 41.422 | 44.985 | 48.231 | 52.190 | 55.000 |
| 32 | 15.134 | 16.362 | 18.291 | 20.072 | 22.271 | 42.585 | 46.194 | 49.480 | 53.486 | 56.328 |
| 33 | 15.814 | 17.073 | 19.046 | 20.866 | 23.110 | 43.745 | 47.400 | 50.724 | 54.774 | 57.646 |
| 34 | 16.501 | 17.789 | 19.806 | 21.664 | 23.952 | 44.903 | 48.602 | 51.966 | 56.061 | 58.964 |
| 35 | 17.191 | 18.508 | 20.569 | 22.465 | 24.796 | 46.059 | 49.802 | 53.203 | 57.340 | 60.272 |
| 36 | 17.887 | 19.233 | 21.336 | 23.269 | 25.643 | 47.212 | 50.998 | 54.437 | 58.619 | 61.581 |
| 37 | 18.584 | 19.960 | 22.105 | 24.075 | 26.492 | 48.363 | 52.192 | 55.667 | 59.891 | 62.880 |
| 38 | 19.289 | 20.691 | 22.878 | 24.884 | 27.343 | 49.513 | 53.384 | 56.896 | 61.162 | 64.181 |
| 39 | 19.994 | 21.425 | 23.654 | 25.695 | 28.196 | 50.660 | 54.572 | 58.119 | 62.426 | 65.473 |
| 40 | 20.706 | 22.164 | 24.433 | 26.509 | 29.050 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |

For $\nu > 40$, $\chi^2_{\alpha,\nu} \approx \nu\left(1 - \frac{2}{9\nu} + z_\alpha\sqrt{\frac{2}{9\nu}}\right)^3$

**Table A.8 t Curve Tail Areas**




| t \ ν | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 |
| 0.1 | .468 | .465 | .463 | .463 | .462 | .462 | .462 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .461 |
| 0.2 | .437 | .430 | .427 | .426 | .425 | .424 | .424 | .423 | .423 | .423 | .423 | .422 | .422 | .422 | .422 | .422 | .422 | .422 |
| 0.3 | .407 | .396 | .392 | .390 | .388 | .387 | .386 | .386 | .386 | .385 | .385 | .385 | .384 | .384 | .384 | .384 | .384 | .384 |
| 0.4 | .379 | .364 | .358 | .355 | .353 | .352 | .351 | .350 | .349 | .349 | .348 | .348 | .348 | .347 | .347 | .347 | .347 | .347 |
| 0.5 | .352 | .333 | .326 | .322 | .319 | .317 | .316 | .315 | .315 | .314 | .313 | .313 | .313 | .312 | .312 | .312 | .312 | .312 |
| 0.6 | .328 | .305 | .295 | .290 | .287 | .285 | .284 | .283 | .282 | .281 | .280 | .280 | .279 | .279 | .279 | .275 | .278 | .278 |
| 0.7 | .306 | .278 | .267 | .261 | .258 | .255 | .253 | .252 | .251 | .250 | .249 | .249 | .248 | .247 | .247 | .247 | .247 | .246 |
| 0.8 | .285 | .254 | .241 | .234 | .230 | .227 | .225 | .223 | .222 | .221 | .220 | .220 | .219 | .218 | .218 | .218 | .217 | .217 |
| 0.9 | .267 | .232 | .217 | .210 | .205 | .201 | .199 | .197 | .196 | .195 | .194 | .193 | .192 | .191 | .191 | .191 | .190 | .190 |
| 1.0 | .250 | .211 | .196 | .187 | .182 | .178 | .175 | .173 | .172 | .170 | .169 | .169 | .168 | .167 | .167 | .166 | .166 | .165 |
| 1.1 | .235 | .193 | .176 | .167 | .162 | .157 | .154 | .152 | .150 | .149 | .147 | .146 | .146 | .144 | .144 | .144 | .143 | .143 |
| 1.2 | .221 | .177 | .158 | .148 | .142 | .138 | .135 | .132 | .130 | .129 | .128 | .127 | .126 | .124 | .124 | .124 | .123 | .123 |
| 1.3 | .209 | .162 | .142 | .132 | .125 | .121 | .117 | .115 | .113 | .111 | .110 | .109 | .108 | .107 | .107 | .106 | .105 | .105 |
| 1.4 | .197 | .148 | .128 | .117 | .110 | .106 | .102 | .100 | .098 | .096 | .095 | .093 | .092 | .091 | .091 | .090 | .090 | .089 |
| 1.5 | .187 | .136 | .115 | .104 | .097 | .092 | .089 | .086 | .084 | .082 | .081 | .080 | .079 | .077 | .077 | .077 | .076 | .075 |
| 1.6 | .178 | .125 | .104 | .092 | .085 | .080 | .077 | .074 | .072 | .070 | .069 | .068 | .067 | .065 | .065 | .065 | .064 | .064 |
| 1.7 | .169 | .116 | .094 | .082 | .075 | .070 | .065 | .064 | .062 | .060 | .059 | .057 | .056 | .055 | .055 | .054 | .054 | .053 |
| 1.8 | .161 | .107 | .085 | .073 | .066 | .061 | .057 | .055 | .053 | .051 | .050 | .049 | .048 | .046 | .046 | .045 | .045 | .044 |
| 1.9 | .154 | .099 | .077 | .065 | .058 | .053 | .050 | .047 | .045 | .043 | .042 | .041 | .040 | .038 | .038 | .038 | .037 | .037 |
| 2.0 | .148 | .092 | .070 | .058 | .051 | .046 | .043 | .040 | .038 | .037 | .035 | .034 | .033 | .032 | .032 | .031 | .031 | .030 |
| 2.1 | .141 | .085 | .063 | .052 | .045 | .040 | .037 | .034 | .033 | .031 | .030 | .029 | .028 | .027 | .027 | .026 | .025 | .025 |
| 2.2 | .136 | .079 | .058 | .046 | .040 | .035 | .032 | .029 | .028 | .026 | .025 | .024 | .023 | .022 | .022 | .021 | .021 | .021 |
| 2.3 | .131 | .074 | .052 | .041 | .035 | .031 | .027 | .025 | .023 | .022 | .021 | .020 | .019 | .018 | .018 | .018 | .017 | .017 |
| 2.4 | .126 | .069 | .048 | .037 | .031 | .027 | .024 | .022 | .020 | .019 | .018 | .017 | .016 | .015 | .015 | .014 | .014 | .014 |
| 2.5 | .121 | .065 | .044 | .033 | .027 | .023 | .020 | .018 | .017 | .016 | .015 | .014 | .013 | .012 | .012 | .012 | .011 | .011 |
| 2.6 | .117 | .061 | .040 | .030 | .024 | .020 | .018 | .016 | .014 | .013 | .012 | .012 | .011 | .010 | .010 | .010 | .009 | .009 |
| 2.7 | .113 | .057 | .037 | .027 | .021 | .018 | .015 | .014 | .012 | .011 | .010 | .010 | .009 | .008 | .008 | .008 | .008 | .007 |
| 2.8 | .109 | .054 | .034 | .024 | .019 | .016 | .013 | .012 | .010 | .009 | .009 | .008 | .008 | .007 | .007 | .006 | .006 | .006 |
| 2.9 | .106 | .051 | .031 | .022 | .017 | .014 | .011 | .010 | .009 | .008 | .007 | .007 | .006 | .005 | .005 | .005 | .005 | .005 |
| 3.0 | .102 | .048 | .029 | .020 | .015 | .012 | .010 | .009 | .007 | .007 | .006 | .006 | .005 | .004 | .004 | .004 | .004 | .004 |
| 3.1 | .099 | .045 | .027 | .018 | .013 | .011 | .009 | .007 | .006 | .006 | .005 | .005 | .004 | .004 | .004 | .003 | .003 | .003 |
| 3.2 | .096 | .043 | .025 | .016 | .012 | .009 | .008 | .006 | .005 | .005 | .004 | .004 | .003 | .003 | .003 | .003 | .003 | .002 |
| 3.3 | .094 | .040 | .023 | .015 | .011 | .008 | .007 | .005 | .005 | .004 | .004 | .003 | .003 | .002 | .002 | .002 | .002 | .002 |
| 3.4 | .091 | .038 | .021 | .014 | .010 | .007 | .006 | .005 | .004 | .003 | .003 | .003 | .002 | .002 | .002 | .002 | .002 | .002 |
| 3.5 | .089 | .036 | .020 | .012 | .009 | .006 | .005 | .004 | .003 | .003 | .002 | .002 | .002 | .002 | .002 | .001 | .001 | .001 |
| 3.6 | .086 | .035 | .018 | .011 | .008 | .006 | .004 | .004 | .003 | .002 | .002 | .002 | .002 | .001 | .001 | .001 | .001 | .001 |
| 3.7 | .084 | .033 | .017 | .010 | .007 | .005 | .004 | .003 | .002 | .002 | .002 | .002 | .001 | .001 | .001 | .001 | .001 | .001 |
| 3.8 | .082 | .031 | .016 | .010 | .006 | .004 | .003 | .003 | .002 | .002 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 |
| 3.9 | .080 | .030 | .015 | .009 | .006 | .004 | .003 | .002 | .002 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 |
| 4.0 | .078 | .029 | .014 | .008 | .005 | .004 | .003 | .002 | .002 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .000 | .000 |

*(continued)*



**Table A.8 t Curve Tail Areas (cont.)**

| t \ ν | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 35 | 40 | 60 | 120 | ∞(= z) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 | .500 |
| 0.1 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .461 | .460 | .460 | .460 | .460 | .460 |
| 0.2 | .422 | .422 | .422 | .422 | .422 | .422 | .422 | .422 | .421 | .421 | .421 | .421 | .421 | .421 | .421 | .421 | .421 |
| 0.3 | .384 | .384 | .384 | .383 | .383 | .383 | .383 | .383 | .383 | .383 | .383 | .383 | .383 | .383 | .383 | .382 | .382 |
| 0.4 | .347 | .347 | .347 | .347 | .346 | .346 | .346 | .346 | .346 | .346 | .346 | .346 | .346 | .346 | .345 | .345 | .345 |
| 0.5 | .311 | .311 | .311 | .311 | .311 | .311 | .311 | .311 | .311 | .310 | .310 | .310 | .310 | .310 | .309 | .309 | .309 |
| 0.6 | .278 | .278 | .278 | .277 | .277 | .277 | .277 | .277 | .277 | .277 | .277 | .277 | .276 | .276 | .275 | .275 | .274 |
| 0.7 | .246 | .246 | .246 | .246 | .245 | .245 | .245 | .245 | .245 | .245 | .245 | .245 | .244 | .244 | .243 | .243 | .242 |
| 0.8 | .217 | .217 | .216 | .216 | .216 | .216 | .216 | .215 | .215 | .215 | .215 | .215 | .215 | .214 | .213 | .213 | .212 |
| 0.9 | .190 | .189 | .189 | .189 | .189 | .189 | .188 | .188 | .188 | .188 | .188 | .188 | .187 | .187 | .186 | .185 | .184 |
| 1.0 | .165 | .165 | .164 | .164 | .164 | .164 | .163 | .163 | .163 | .163 | .163 | .163 | .162 | .162 | .161 | .160 | .159 |
| 1.1 | .143 | .142 | .142 | .142 | .141 | .141 | .141 | .141 | .141 | .140 | .140 | .140 | .139 | .139 | .138 | .137 | .136 |
| 1.2 | .122 | .122 | .122 | .121 | .121 | .121 | .121 | .120 | .120 | .120 | .120 | .120 | .119 | .119 | .117 | .116 | .115 |
| 1.3 | .105 | .104 | .104 | .104 | .103 | .103 | .103 | .103 | .102 | .102 | .102 | .102 | .101 | .101 | .099 | .098 | .097 |
| 1.4 | .089 | .089 | .088 | .088 | .087 | .087 | .087 | .087 | .086 | .086 | .086 | .086 | .085 | .085 | .083 | .082 | .081 |
| 1.5 | .075 | .075 | .074 | .074 | .074 | .073 | .073 | .073 | .073 | .072 | .072 | .072 | .071 | .071 | .069 | .068 | .067 |
| 1.6 | .063 | .063 | .062 | .062 | .062 | .061 | .061 | .061 | .061 | .060 | .060 | .060 | .059 | .059 | .057 | .056 | .055 |
| 1.7 | .053 | .052 | .052 | .052 | .051 | .051 | .051 | .051 | .050 | .050 | .050 | .050 | .049 | .048 | .047 | .046 | .045 |
| 1.8 | .044 | .043 | .043 | .043 | .042 | .042 | .042 | .042 | .042 | .041 | .041 | .041 | .040 | .040 | .038 | .037 | .036 |
| 1.9 | .036 | .036 | .036 | .035 | .035 | .035 | .035 | .034 | .034 | .034 | .034 | .034 | .033 | .032 | .031 | .030 | .029 |
| 2.0 | .030 | .030 | .029 | .029 | .029 | .028 | .028 | .028 | .028 | .028 | .027 | .027 | .027 | .026 | .025 | .024 | .023 |
| 2.1 | .025 | .024 | .024 | .024 | .023 | .023 | .023 | .023 | .023 | .022 | .022 | .022 | .022 | .021 | .020 | .019 | .018 |
| 2.2 | .020 | .020 | .020 | .019 | .019 | .019 | .019 | .018 | .018 | .018 | .018 | .018 | .017 | .017 | .016 | .015 | .014 |
| 2.3 | .016 | .016 | .016 | .016 | .015 | .015 | .015 | .015 | .015 | .015 | .014 | .014 | .014 | .013 | .012 | .012 | .011 |
| 2.4 | .013 | .013 | .013 | .013 | .012 | .012 | .012 | .012 | .012 | .012 | .012 | .011 | .011 | .011 | .010 | .009 | .008 |
| 2.5 | .011 | .011 | .010 | .010 | .010 | .010 | .010 | .010 | .009 | .009 | .009 | .009 | .009 | .008 | .008 | .007 | .006 |
| 2.6 | .009 | .009 | .008 | .008 | .008 | .008 | .008 | .008 | .007 | .007 | .007 | .007 | .007 | .007 | .006 | .005 | .005 |
| 2.7 | .007 | .007 | .007 | .007 | .006 | .006 | .006 | .006 | .006 | .006 | .006 | .006 | .005 | .005 | .004 | .004 | .003 |
| 2.8 | .006 | .006 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .004 | .004 | .004 | .003 | .003 | .003 |
| 2.9 | .005 | .004 | .004 | .004 | .004 | .004 | .004 | .004 | .004 | .004 | .004 | .003 | .003 | .003 | .003 | .002 | .002 |
| 3.0 | .004 | .004 | .003 | .003 | .003 | .003 | .003 | .003 | .003 | .003 | .003 | .003 | .002 | .002 | .002 | .002 | .001 |
| 3.1 | .003 | .003 | .003 | .003 | .003 | .002 | .002 | .002 | .002 | .002 | .002 | .002 | .002 | .002 | .001 | .001 | .001 |
| 3.2 | .002 | .002 | .002 | .002 | .002 | .002 | .002 | .002 | .002 | .002 | .002 | .002 | .001 | .001 | .001 | .001 | .001 |
| 3.3 | .002 | .002 | .002 | .002 | .002 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .000 |
| 3.4 | .002 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .000 | .000 |
| 3.5 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .000 | .000 | .000 |
| 3.6 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .000 | .000 | .000 | .000 | .000 |
| 3.7 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 3.8 | .001 | .001 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 3.9 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 4.0 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |

**Table A.9** Critical Values for $F$ Distributions

$\nu_2$ = denominator df; $\nu_1$ = numerator df

| $\nu_2$ | $\alpha$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .100 | 39.86 | 49.50 | 53.59 | 55.83 | 57.24 | 58.20 | 58.91 | 59.44 | 59.86 |
| | .050 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 |
| | .010 | 4052.2 | 4999.5 | 5403.4 | 5624.6 | 5763.6 | 5859.0 | 5928.4 | 5981.1 | 6022.5 |
| | .001 | 405284 | 500000 | 540379 | 562500 | 576405 | 585937 | 592873 | 598144 | 602284 |
| 2 | .100 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 |
| | .050 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 |
| | .010 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 |
| | .001 | 998.50 | 999.00 | 999.17 | 999.25 | 999.30 | 999.33 | 999.36 | 999.37 | 999.39 |
| 3 | .100 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 |
| | .050 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 |
| | .010 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 |
| | .001 | 167.03 | 148.50 | 141.11 | 137.10 | 134.58 | 132.85 | 131.58 | 130.62 | 129.86 |
| 4 | .100 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.94 |
| | .050 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 |
| | .010 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 |
| | .001 | 74.14 | 61.25 | 56.18 | 53.44 | 51.71 | 50.53 | 49.66 | 49.00 | 48.47 |
| 5 | .100 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 |
| | .050 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 |
| | .010 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 |
| | .001 | 47.18 | 37.12 | 33.20 | 31.09 | 29.75 | 28.83 | 28.16 | 27.65 | 27.24 |
| 6 | .100 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 |
| | .050 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 |
| | .010 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 |
| | .001 | 35.51 | 27.00 | 23.70 | 21.92 | 20.80 | 20.03 | 19.46 | 19.03 | 18.69 |
| 7 | .100 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 |
| | .050 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 |
| | .010 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 |
| | .001 | 29.25 | 21.69 | 18.77 | 17.20 | 16.21 | 15.52 | 15.02 | 14.63 | 14.33 |
| 8 | .100 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 |
| | .050 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 |
| | .010 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 |
| | .001 | 25.41 | 18.49 | 15.83 | 14.39 | 13.48 | 12.86 | 12.40 | 12.05 | 11.77 |
| 9 | .100 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 |
| | .050 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 |
| | .010 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 |
| | .001 | 22.86 | 16.39 | 13.90 | 12.56 | 11.71 | 11.13 | 10.70 | 10.37 | 10.11 |
| 10 | .100 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 |
| | .050 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 |
| | .010 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 |
| | .001 | 21.04 | 14.91 | 12.55 | 11.28 | 10.48 | 9.93 | 9.52 | 9.20 | 8.96 |
| 11 | .100 | 3.23 | 2.86 | 2.66 | 2.54 | 2.45 | 2.39 | 2.34 | 2.30 | 2.27 |
| | .050 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 |
| | .010 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 |
| | .001 | 19.69 | 13.81 | 11.56 | 10.35 | 9.58 | 9.05 | 8.66 | 8.35 | 8.12 |
| 12 | .100 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 |
| | .050 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 |
| | .010 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 |
| | .001 | 18.64 | 12.97 | 10.80 | 9.63 | 8.89 | 8.38 | 8.00 | 7.71 | 7.48 |

(continued)



... Values for $F$ Distributions (cont.)

| $\nu_2$ | $\alpha$ | 10 | 12 | 15 | 20 | 25 | 30 | 40 | 50 | 60 | 120 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .100 | 60.19 | 60.71 | 61.22 | 61.74 | 62.05 | 62.26 | 62.53 | 62.69 | 62.79 | 63.06 | 63.3 |
| | .050 | 241.88 | 243.91 | 245.95 | 248.01 | 249.26 | 250.10 | 251.14 | 251.77 | 252.20 | 253.25 | 254.1 |
| | .010 | 6055.8 | 6106.3 | 6157.3 | 6208.7 | 6239.8 | 6260.6 | 6286.8 | 6302.5 | 6313.0 | 6339.4 | 6362.7 |
| | .001 | 605621 | 610668 | 615764 | 620908 | 624017 | 626099 | 628712 | 630285 | 631337 | 633972 | 63630 |
| 2 | .100 | 9.39 | 9.41 | 9.42 | 9.44 | 9.45 | 9.46 | 9.47 | 9.47 | 9.47 | 9.48 | 9.4 |
| | .050 | 19.40 | 19.41 | 19.43 | 19.45 | 19.46 | 19.46 | 19.47 | 19.48 | 19.48 | 19.49 | 19.4 |
| | .010 | 99.40 | 99.42 | 99.43 | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.48 | 99.49 | 99.5 |
| | .001 | 999.40 | 999.42 | 999.43 | 999.45 | 999.46 | 999.47 | 999.47 | 999.48 | 999.48 | 999.49 | 999.5 |
| 3 | .100 | 5.23 | 5.22 | 5.20 | 5.18 | 5.17 | 5.17 | 5.16 | 5.15 | 5.15 | 5.14 | 5.1 |
| | .050 | 8.79 | 8.74 | 8.70 | 8.66 | 8.63 | 8.62 | 8.59 | 8.58 | 8.57 | 8.55 | 8.5 |
| | .010 | 27.23 | 27.05 | 26.87 | 26.69 | 26.58 | 26.50 | 26.41 | 26.35 | 26.32 | 26.22 | 26.1 |
| | .001 | 129.25 | 128.32 | 127.37 | 126.42 | 125.84 | 125.45 | 124.96 | 124.66 | 124.47 | 123.97 | 123.5 |
| 4 | .100 | 3.92 | 3.90 | 3.87 | 3.84 | 3.83 | 3.82 | 3.80 | 3.80 | 3.79 | 3.78 | 3.7 |
| | .050 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.70 | 5.69 | 5.66 | 5.6 |
| | .010 | 14.55 | 14.37 | 14.20 | 14.02 | 13.91 | 13.84 | 13.75 | 13.69 | 13.65 | 13.56 | 13.4 |
| | .001 | 48.05 | 47.41 | 46.76 | 46.10 | 45.70 | 45.43 | 45.09 | 44.88 | 44.75 | 44.40 | 44.0 |
| 5 | .100 | 3.30 | 3.27 | 3.24 | 3.21 | 3.19 | 3.17 | 3.16 | 3.15 | 3.14 | 3.12 | 3.1 |
| | .050 | 4.74 | 4.68 | 4.62 | 4.56 | 4.52 | 4.50 | 4.46 | 4.44 | 4.43 | 4.40 | 4.3 |
| | .010 | 10.05 | 9.89 | 9.72 | 9.55 | 9.45 | 9.38 | 9.29 | 9.24 | 9.20 | 9.11 | 9.0 |
| | .001 | 26.92 | 26.42 | 25.91 | 25.39 | 25.08 | 24.87 | 24.60 | 24.44 | 24.33 | 24.06 | 23.8 |
| 6 | .100 | 2.94 | 2.90 | 2.87 | 2.84 | 2.81 | 2.80 | 2.78 | 2.77 | 2.76 | 2.74 | 2.7 |
| | .050 | 4.06 | 4.00 | 3.94 | 3.87 | 3.83 | 3.81 | 3.77 | 3.75 | 3.74 | 3.70 | 3.6 |
| | .010 | 7.87 | 7.72 | 7.56 | 7.40 | 7.30 | 7.23 | 7.14 | 7.09 | 7.06 | 6.97 | 6.8 |
| | .001 | 18.41 | 17.99 | 17.56 | 17.12 | 16.85 | 16.67 | 16.44 | 16.31 | 16.21 | 15.98 | 15.7 |
| 7 | .100 | 2.70 | 2.67 | 2.63 | 2.59 | 2.57 | 2.56 | 2.54 | 2.52 | 2.51 | 2.49 | 2.4 |
| | .050 | 3.64 | 3.57 | 3.51 | 3.44 | 3.40 | 3.38 | 3.34 | 3.32 | 3.30 | 3.27 | 3.2 |
| | .010 | 6.62 | 6.47 | 6.31 | 6.16 | 6.06 | 5.99 | 5.91 | 5.86 | 5.82 | 5.74 | 5.6 |
| | .001 | 14.08 | 13.71 | 13.32 | 12.93 | 12.69 | 12.53 | 12.33 | 12.20 | 12.12 | 11.91 | 11.7 |
| 8 | .100 | 2.54 | 2.50 | 2.46 | 2.42 | 2.40 | 2.38 | 2.36 | 2.35 | 2.34 | 2.32 | 2.3 |
| | .050 | 3.35 | 3.28 | 3.22 | 3.15 | 3.11 | 3.08 | 3.04 | 3.02 | 3.01 | 2.97 | 2.9 |
| | .010 | 5.81 | 5.67 | 5.52 | 5.36 | 5.26 | 5.20 | 5.12 | 5.07 | 5.03 | 4.95 | 4.87 |
| | .001 | 11.54 | 11.19 | 10.84 | 10.48 | 10.26 | 10.11 | 9.92 | 9.80 | 9.73 | 9.53 | 9.3 |
| 9 | .100 | 2.42 | 2.38 | 2.34 | 2.30 | 2.27 | 2.25 | 2.23 | 2.22 | 2.21 | 2.18 | 2.1 |
| | .050 | 3.14 | 3.07 | 3.01 | 2.94 | 2.89 | 2.86 | 2.83 | 2.80 | 2.79 | 2.75 | 2.71 |
| | .010 | 5.26 | 5.11 | 4.96 | 4.81 | 4.71 | 4.65 | 4.57 | 4.52 | 4.48 | 4.40 | 4.3 |
| | .001 | 9.89 | 9.57 | 9.24 | 8.90 | 8.69 | 8.55 | 8.37 | 8.26 | 8.19 | 8.00 | 7.84 |
| 10 | .100 | 2.32 | 2.28 | 2.24 | 2.20 | 2.17 | 2.16 | 2.13 | 2.12 | 2.11 | 2.08 | 2.06 |
| | .050 | 2.98 | 2.91 | 2.85 | 2.77 | 2.73 | 2.70 | 2.66 | 2.64 | 2.62 | 2.58 | 2.54 |
| | .010 | 4.85 | 4.71 | 4.56 | 4.41 | 4.31 | 4.25 | 4.17 | 4.12 | 4.08 | 4.00 | 3.92 |
| | .001 | 8.75 | 8.45 | 8.13 | 7.80 | 7.60 | 7.47 | 7.30 | 7.19 | 7.12 | 6.94 | 6.78 |
| 11 | .100 | 2.25 | 2.21 | 2.17 | 2.12 | 2.10 | 2.08 | 2.05 | 2.04 | 2.03 | 2.00 | 1.98 |
| | .050 | 2.85 | 2.79 | 2.72 | 2.65 | 2.60 | 2.57 | 2.53 | 2.51 | 2.49 | 2.45 | 2.41 |
| | .010 | 4.54 | 4.40 | 4.25 | 4.10 | 4.01 | 3.94 | 3.86 | 3.81 | 3.78 | 3.69 | 3.61 |
| | .001 | 7.92 | 7.63 | 7.32 | 7.01 | 6.81 | 6.68 | 6.52 | 6.42 | 6.35 | 6.18 | 6.02 |
| 12 | .100 | 2.19 | 2.15 | 2.10 | 2.06 | 2.03 | 2.01 | 1.99 | 1.97 | 1.96 | 1.93 | 1.91 |
| | .050 | 2.75 | 2.69 | 2.62 | 2.54 | 2.50 | 2.47 | 2.43 | 2.40 | 2.38 | 2.34 | 2.30 |
| | .010 | 4.30 | 4.16 | 4.01 | 3.86 | 3.76 | 3.70 | 3.62 | 3.57 | 3.54 | 3.45 | 3.37 |
| | .001 | 7.29 | 7.00 | 6.71 | 6.40 | 6.22 | 6.09 | 5.93 | 5.83 | 5.76 | 5.59 | 5.44 |

(continued)

Critical Values for F Distributions (cont.)

$\nu_1$ = numerator df; $\nu_2$ = denominator df

| $\nu_2$ | α | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 13 | .100 | 3.14 | 2.76 | 2.56 | 2.43 | 2.35 | 2.28 | 2.23 | 2.20 | 2.16 |
| | .050 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 |
| | .010 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 |
| | .001 | 17.82 | 12.31 | 10.21 | 9.07 | 8.35 | 7.86 | 7.49 | 7.21 | 6.98 |
| 14 | .100 | 3.10 | 2.73 | 2.52 | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.12 |
| | .050 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 |
| | .010 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 |
| | .001 | 17.14 | 11.78 | 9.73 | 8.62 | 7.92 | 7.44 | 7.08 | 6.80 | 6.58 |
| 15 | .100 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 |
| | .050 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 |
| | .010 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 |
| | .001 | 16.59 | 11.34 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 |
| 16 | .100 | 3.05 | 2.67 | 2.46 | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.06 |
| | .050 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 |
| | .010 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 |
| | .001 | 16.12 | 10.97 | 9.01 | 7.94 | 7.27 | 6.80 | 6.46 | 6.19 | 5.98 |
| 17 | .100 | 3.03 | 2.64 | 2.44 | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.03 |
| | .050 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 |
| | .010 | 8.40 | 6.11 | 5.19 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 |
| | .001 | 15.72 | 10.66 | 8.73 | 7.68 | 7.02 | 6.56 | 6.22 | 5.96 | 5.75 |
| 18 | .100 | 3.01 | 2.62 | 2.42 | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 |
| | .050 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 |
| | .010 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 |
| | .001 | 15.38 | 10.39 | 8.49 | 7.46 | 6.81 | 6.35 | 6.02 | 5.76 | 5.56 |
| 19 | .100 | 2.99 | 2.61 | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 |
| | .050 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 |
| | .010 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 |
| | .001 | 15.08 | 10.16 | 8.28 | 7.27 | 6.62 | 6.18 | 5.85 | 5.59 | 5.39 |
| 20 | .100 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 |
| | .050 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
| | .010 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
| | .001 | 14.82 | 9.95 | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 |
| 21 | .100 | 2.96 | 2.57 | 2.36 | 2.23 | 2.14 | 2.08 | 2.02 | 1.98 | 1.95 |
| | .050 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 |
| | .010 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 |
| | .001 | 14.59 | 9.77 | 7.94 | 6.95 | 6.32 | 5.88 | 5.56 | 5.31 | 5.11 |
| 22 | .100 | 2.95 | 2.56 | 2.35 | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 |
| | .050 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 |
| | .010 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 |
| | .001 | 14.38 | 9.61 | 7.80 | 6.81 | 6.19 | 5.76 | 5.44 | 5.19 | 4.99 |
| 23 | .100 | 2.94 | 2.55 | 2.34 | 2.21 | 2.11 | 2.05 | 1.99 | 1.95 | 1.92 |
| | .050 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 |
| | .010 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 |
| | .001 | 14.20 | 9.47 | 7.67 | 6.70 | 6.08 | 5.65 | 5.33 | 5.09 | 4.89 |
| 24 | .100 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 |
| | .050 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |
| | .010 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |
| | .001 | 14.03 | 9.34 | 7.55 | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 |

(continued)

**Table A.9** Critical Values for F Distributions (cont.)

$\nu_1$ = numerator df

| $\nu_2$ | α | 10 | 12 | 15 | 20 | 25 | 30 | 40 | 50 | 60 | 120 | 1000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 | .100 | 2.14 | 2.10 | 2.05 | 2.01 | 1.98 | 1.96 | 1.93 | 1.92 | 1.90 | [illegible] | [illegible] |
| | .050 | 2.67 | 2.60 | 2.53 | 2.46 | 2.41 | 2.38 | 2.34 | 2.31 | 2.30 | [illegible] | [illegible] |
| | .010 | 4.10 | 3.96 | 3.82 | 3.66 | 3.57 | 3.51 | 3.43 | 3.38 | 3.34 | [illegible] | [illegible] |
| | .001 | 6.80 | 6.52 | 6.23 | 5.93 | 5.75 | 5.63 | 5.47 | 5.37 | 5.30 | [illegible] | [illegible] |
| 14 | .100 | 2.10 | 2.05 | 2.01 | 1.96 | 1.93 | 1.91 | 1.89 | 1.87 | 1.86 | [illegible] | [illegible] |
| | .050 | 2.60 | 2.53 | 2.46 | 2.39 | 2.34 | 2.31 | 2.27 | 2.24 | 2.22 | [illegible] | [illegible] |
| | .010 | 3.94 | 3.80 | 3.66 | 3.51 | 3.41 | 3.35 | 3.27 | 3.22 | 3.18 | [illegible] | [illegible] |
| | .001 | 6.40 | 6.13 | 5.85 | 5.56 | 5.38 | 5.25 | 5.10 | 5.00 | 4.94 | [illegible] | [illegible] |
| 15 | .100 | 2.06 | 2.02 | 1.97 | 1.92 | 1.89 | 1.87 | 1.85 | 1.83 | 1.82 | [illegible] | [illegible] |
| | .050 | 2.54 | 2.48 | 2.40 | 2.33 | 2.28 | 2.25 | 2.20 | 2.18 | 2.16 | [illegible] | [illegible] |
| | .010 | 3.80 | 3.67 | 3.52 | 3.37 | 3.28 | 3.21 | 3.13 | 3.08 | 3.05 | [illegible] | [illegible] |
| | .001 | 6.08 | 5.81 | 5.54 | 5.25 | 5.07 | 4.95 | 4.80 | 4.70 | 4.64 | [illegible] | [illegible] |
| 16 | .100 | 2.03 | 1.99 | 1.94 | 1.89 | 1.86 | 1.84 | 1.81 | 1.79 | 1.78 | [illegible] | [illegible] |
| | .050 | 2.49 | 2.42 | 2.35 | 2.28 | 2.23 | 2.19 | 2.15 | 2.12 | 2.11 | [illegible] | [illegible] |
| | .010 | 3.69 | 3.55 | 3.41 | 3.26 | 3.16 | 3.10 | 3.02 | 2.97 | 2.93 | [illegible] | [illegible] |
| | .001 | 5.81 | 5.55 | 5.27 | 4.99 | 4.82 | 4.70 | 4.54 | 4.45 | 4.39 | [illegible] | [illegible] |
| 17 | .100 | 2.00 | 1.96 | 1.91 | 1.86 | 1.83 | 1.81 | 1.78 | 1.76 | 1.75 | [illegible] | [illegible] |
| | .050 | 2.45 | 2.38 | 2.31 | 2.23 | 2.18 | 2.15 | 2.10 | 2.08 | 2.06 | [illegible] | [illegible] |
| | .010 | 3.59 | 3.46 | 3.31 | 3.16 | 3.07 | 3.00 | 2.92 | 2.87 | 2.83 | [illegible] | [illegible] |
| | .001 | 5.58 | 5.32 | 5.05 | 4.78 | 4.60 | 4.48 | 4.33 | 4.24 | 4.18 | [illegible] | [illegible] |
| 18 | .100 | 1.98 | 1.93 | 1.89 | 1.84 | 1.80 | 1.78 | 1.75 | 1.74 | 1.72 | [illegible] | [illegible] |
| | .050 | 2.41 | 2.34 | 2.27 | 2.19 | 2.14 | 2.11 | 2.06 | 2.04 | 2.02 | [illegible] | [illegible] |
| | .010 | 3.51 | 3.37 | 3.23 | 3.08 | 2.98 | 2.92 | 2.84 | 2.78 | 2.75 | 2.66 | [illegible] |
| | .001 | 5.39 | 5.13 | 4.87 | 4.59 | 4.42 | 4.30 | 4.15 | 4.06 | 4.00 | [illegible] | [illegible] |
| 19 | .100 | 1.96 | 1.91 | 1.86 | 1.81 | 1.78 | 1.76 | 1.73 | 1.71 | 1.70 | [illegible] | [illegible] |
| | .050 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 2.00 | 1.98 | [illegible] | [illegible] |
| | .010 | 3.43 | 3.30 | 3.15 | 3.00 | 2.91 | 2.84 | 2.76 | 2.71 | 2.67 | [illegible] | [illegible] |
| | .001 | 5.22 | 4.97 | 4.70 | 4.43 | 4.26 | 4.14 | 3.99 | 3.90 | 3.84 | [illegible] | [illegible] |
| 20 | .100 | 1.94 | 1.89 | 1.84 | 1.79 | 1.76 | 1.74 | 1.71 | 1.69 | 1.68 | [illegible] | [illegible] |
| | .050 | 2.35 | 2.28 | 2.20 | 2.12 | 2.07 | 2.04 | 1.99 | 1.97 | 1.95 | [illegible] | [illegible] |
| | .010 | 3.37 | 3.23 | 3.09 | 2.94 | 2.84 | 2.78 | 2.69 | 2.64 | 2.61 | 2.52 | [illegible] |
| | .001 | 5.08 | 4.82 | 4.56 | 4.29 | 4.12 | 4.00 | 3.86 | 3.77 | 3.70 | [illegible] | [illegible] |
| 21 | .100 | 1.92 | 1.87 | 1.83 | 1.78 | 1.74 | 1.72 | 1.69 | 1.67 | 1.66 | [illegible] | [illegible] |
| | .050 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.94 | 1.92 | [illegible] | [illegible] |
| | .010 | 3.31 | 3.17 | 3.03 | 2.88 | 2.79 | 2.72 | 2.64 | 2.58 | 2.55 | [illegible] | [illegible] |
| | .001 | 4.95 | 4.70 | 4.44 | 4.17 | 4.00 | 3.88 | 3.74 | 3.64 | 3.58 | 3.42 | [illegible] |
| 22 | .100 | 1.90 | 1.86 | 1.81 | 1.76 | 1.73 | 1.70 | 1.67 | 1.65 | 1.64 | [illegible] | [illegible] |
| | .050 | 2.30 | 2.23 | 2.15 | 2.07 | 2.02 | 1.98 | 1.94 | 1.91 | 1.89 | [illegible] | [illegible] |
| | .010 | 3.26 | 3.12 | 2.98 | 2.83 | 2.73 | 2.67 | 2.58 | 2.53 | 2.50 | [illegible] | [illegible] |
| | .001 | 4.83 | 4.58 | 4.33 | 4.06 | 3.89 | 3.78 | 3.63 | 3.54 | 3.48 | [illegible] | [illegible] |
| 23 | .100 | 1.89 | 1.84 | 1.80 | 1.74 | 1.71 | 1.69 | 1.66 | 1.64 | 1.62 | [illegible] | [illegible] |
| | .050 | 2.27 | 2.20 | 2.13 | 2.05 | 2.00 | 1.96 | 1.91 | 1.88 | 1.86 | [illegible] | [illegible] |
| | .010 | 3.21 | 3.07 | 2.93 | 2.78 | 2.69 | 2.62 | 2.54 | 2.48 | 2.45 | [illegible] | [illegible] |
| | .001 | 4.73 | 4.48 | 4.23 | 3.96 | 3.79 | 3.68 | 3.53 | 3.44 | 3.38 | [illegible] | [illegible] |
| 24 | .100 | 1.88 | 1.83 | 1.78 | 1.73 | 1.70 | 1.67 | 1.64 | 1.62 | 1.61 | [illegible] | [illegible] |
| | .050 | 2.25 | 2.18 | 2.11 | 2.03 | 1.97 | 1.94 | 1.89 | 1.86 | 1.84 | [illegible] | [illegible] |
| | .010 | 3.17 | 3.03 | 2.89 | 2.74 | 2.64 | 2.58 | 2.49 | 2.44 | 2.40 | [illegible] | [illegible] |
| | .001 | 4.64 | 4.39 | 4.14 | 3.87 | 3.71 | 3.59 | 3.45 | 3.36 | 3.29 | [illegible] | [illegible] |

(continued)

**Table A.9** Critical Values for $F$ Distributions *(cont.)*

$\nu_2$ = denominator df; $\nu_1$ = numerator df

| $\nu_2$ | $\alpha$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 25 | .100 | 2.92 | 2.53 | 2.32 | 2.18 | 2.09 | 2.02 | 1.97 | 1.93 | 1.89 |
| | .050 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 |
| | .010 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 |
| | .001 | 13.88 | 9.22 | 7.45 | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 |
| 26 | .100 | 2.91 | 2.52 | 2.31 | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 |
| | .050 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 |
| | .010 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 |
| | .001 | 13.74 | 9.12 | 7.36 | 6.41 | 5.80 | 5.38 | 5.07 | 4.83 | 4.64 |
| 27 | .100 | 2.90 | 2.51 | 2.30 | 2.17 | 2.07 | 2.00 | 1.95 | 1.91 | 1.87 |
| | .050 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 |
| | .010 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 |
| | .001 | 13.61 | 9.02 | 7.27 | 6.33 | 5.73 | 5.31 | 5.00 | 4.76 | 4.57 |
| 28 | .100 | 2.89 | 2.50 | 2.29 | 2.16 | 2.06 | 2.00 | 1.94 | 1.90 | 1.87 |
| | .050 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 |
| | .010 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 |
| | .001 | 13.50 | 8.93 | 7.19 | 6.25 | 5.66 | 5.24 | 4.93 | 4.69 | 4.50 |
| 29 | .100 | 2.89 | 2.50 | 2.28 | 2.15 | 2.06 | 1.99 | 1.93 | 1.89 | 1.86 |
| | .050 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 |
| | .010 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 |
| | .001 | 13.39 | 8.85 | 7.12 | 6.19 | 5.59 | 5.18 | 4.87 | 4.64 | 4.45 |
| 30 | .100 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 |
| | .050 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 |
| | .010 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 |
| | .001 | 13.29 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 |
| 40 | .100 | 2.84 | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 |
| | .050 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 |
| | .010 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 |
| | .001 | 12.61 | 8.25 | 6.59 | 5.70 | 5.13 | 4.73 | 4.44 | 4.21 | 4.02 |
| 50 | .100 | 2.81 | 2.41 | 2.20 | 2.06 | 1.97 | 1.90 | 1.84 | 1.80 | 1.76 |
| | .050 | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 |
| | .010 | 7.17 | 5.06 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 |
| | .001 | 12.22 | 7.96 | 6.34 | 5.46 | 4.90 | 4.51 | 4.22 | 4.00 | 3.82 |
| 60 | .100 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 |
| | .050 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 |
| | .010 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 |
| | .001 | 11.97 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 |
| 100 | .100 | 2.76 | 2.36 | 2.14 | 2.00 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 |
| | .050 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 |
| | .010 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 |
| | .001 | 11.50 | 7.41 | 5.86 | 5.02 | 4.48 | 4.11 | 3.83 | 3.61 | 3.44 |
| 200 | .100 | 2.73 | 2.33 | 2.11 | 1.97 | 1.88 | 1.80 | 1.75 | 1.70 | 1.66 |
| | .050 | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 |
| | .010 | 6.76 | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 |
| | .001 | 11.15 | 7.15 | 5.63 | 4.81 | 4.29 | 3.92 | 3.65 | 3.43 | 3.26 |
| 1000 | .100 | 2.71 | 2.31 | 2.09 | 1.95 | 1.85 | 1.78 | 1.72 | 1.68 | 1.64 |
| | .050 | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 |
| | .010 | 6.66 | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 |
| | .001 | 10.89 | 6.96 | 5.46 | 4.65 | 4.14 | 3.78 | 3.51 | 3.30 | 3.13 |

*(continued)*



**Table A.9** Critical Values for $F$ Distributions *(cont.)*

$\nu_1$ = numerator df

| $\nu_2$ | $\alpha$ | 10 | 12 | 15 | 20 | 25 | 30 | 40 | 50 | 60 | 120 | 1000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | .100 | 1.87 | 1.82 | 1.77 | 1.72 | 1.68 | 1.66 | 1.63 | 1.61 | 1.59 | 1.56 | 1.52 |
| | .050 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.84 | 1.82 | 1.77 | 1.72 |
| | .010 | 3.13 | 2.99 | 2.85 | 2.70 | 2.60 | 2.54 | 2.45 | 2.40 | 2.36 | 2.27 | 2.18 |
| | .001 | 4.56 | 4.31 | 4.06 | 3.79 | 3.63 | 3.52 | 3.37 | 3.28 | 3.22 | 3.06 | 2.91 |
| 26 | .100 | 1.86 | 1.81 | 1.76 | 1.71 | 1.67 | 1.65 | 1.61 | 1.59 | 1.58 | 1.54 | 1.51 |
| | .050 | 2.22 | 2.15 | 2.07 | 1.99 | 1.94 | 1.90 | 1.85 | 1.82 | 1.80 | 1.75 | 1.70 |
| | .010 | 3.09 | 2.96 | 2.81 | 2.66 | 2.57 | 2.50 | 2.42 | 2.36 | 2.33 | 2.23 | 2.14 |
| | .001 | 4.48 | 4.24 | 3.99 | 3.72 | 3.56 | 3.44 | 3.30 | 3.21 | 3.15 | 2.99 | 2.84 |
| 27 | .100 | 1.85 | 1.80 | 1.75 | 1.70 | 1.66 | 1.64 | 1.60 | 1.58 | 1.57 | 1.53 | 1.50 |
| | .050 | 2.20 | 2.13 | 2.06 | 1.97 | 1.92 | 1.88 | 1.84 | 1.81 | 1.79 | 1.73 | 1.68 |
| | .010 | 3.06 | 2.93 | 2.78 | 2.63 | 2.54 | 2.47 | 2.38 | 2.33 | 2.29 | 2.20 | 2.11 |
| | .001 | 4.41 | 4.17 | 3.92 | 3.66 | 3.49 | 3.38 | 3.23 | 3.14 | 3.08 | 2.92 | 2.78 |
| 28 | .100 | 1.84 | 1.79 | 1.74 | 1.69 | 1.65 | 1.63 | 1.59 | 1.57 | 1.56 | 1.52 | 1.48 |
| | .050 | 2.19 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.79 | 1.77 | 1.71 | 1.66 |
| | .010 | 3.03 | 2.90 | 2.75 | 2.60 | 2.51 | 2.44 | 2.35 | 2.30 | 2.26 | 2.17 | 2.08 |
| | .001 | 4.35 | 4.11 | 3.86 | 3.60 | 3.43 | 3.32 | 3.18 | 3.09 | 3.02 | 2.86 | 2.72 |
| 29 | .100 | 1.83 | 1.78 | 1.73 | 1.68 | 1.64 | 1.62 | 1.58 | 1.56 | 1.55 | 1.51 | 1.47 |
| | .050 | 2.18 | 2.10 | 2.03 | 1.94 | 1.89 | 1.85 | 1.81 | 1.77 | 1.75 | 1.70 | 1.65 |
| | .010 | 3.00 | 2.87 | 2.73 | 2.57 | 2.48 | 2.41 | 2.33 | 2.27 | 2.23 | 2.14 | 2.05 |
| | .001 | 4.29 | 4.05 | 3.80 | 3.54 | 3.38 | 3.27 | 3.12 | 3.03 | 2.97 | 2.81 | 2.66 |
| 30 | .100 | 1.82 | 1.77 | 1.72 | 1.67 | 1.63 | 1.61 | 1.57 | 1.55 | 1.54 | 1.50 | 1.46 |
| | .050 | 2.16 | 2.09 | 2.01 | 1.93 | 1.88 | 1.84 | 1.79 | 1.76 | 1.74 | 1.68 | 1.63 |
| | .010 | 2.98 | 2.84 | 2.70 | 2.55 | 2.45 | 2.39 | 2.30 | 2.25 | 2.21 | 2.11 | 2.02 |
| | .001 | 4.24 | 4.00 | 3.75 | 3.49 | 3.33 | 3.22 | 3.07 | 2.98 | 2.92 | 2.76 | 2.61 |
| 40 | .100 | 1.76 | 1.71 | 1.66 | 1.61 | 1.57 | 1.54 | 1.51 | 1.48 | 1.47 | 1.42 | 1.38 |
| | .050 | 2.08 | 2.00 | 1.92 | 1.84 | 1.78 | 1.74 | 1.69 | 1.66 | 1.64 | 1.58 | 1.52 |
| | .010 | 2.80 | 2.66 | 2.52 | 2.37 | 2.27 | 2.20 | 2.11 | 2.06 | 2.02 | 1.92 | 1.82 |
| | .001 | 3.87 | 3.64 | 3.40 | 3.14 | 2.98 | 2.87 | 2.73 | 2.64 | 2.57 | 2.41 | 2.25 |
| 50 | .100 | 1.73 | 1.68 | 1.63 | 1.57 | 1.53 | 1.50 | 1.46 | 1.44 | 1.42 | 1.38 | 1.33 |
| | .050 | 2.03 | 1.95 | 1.87 | 1.78 | 1.73 | 1.69 | 1.63 | 1.60 | 1.58 | 1.51 | 1.45 |
| | .010 | 2.70 | 2.56 | 2.42 | 2.27 | 2.17 | 2.10 | 2.01 | 1.95 | 1.91 | 1.80 | 1.70 |
| | .001 | 3.67 | 3.44 | 3.20 | 2.95 | 2.79 | 2.68 | 2.53 | 2.44 | 2.38 | 2.21 | 2.05 |
| 60 | .100 | 1.71 | 1.66 | 1.60 | 1.54 | 1.50 | 1.48 | 1.44 | 1.41 | 1.40 | 1.35 | 1.30 |
| | .050 | 1.99 | 1.92 | 1.84 | 1.75 | 1.69 | 1.65 | 1.59 | 1.56 | 1.53 | 1.47 | 1.40 |
| | .010 | 2.63 | 2.50 | 2.35 | 2.20 | 2.10 | 2.03 | 1.94 | 1.88 | 1.84 | 1.73 | 1.62 |
| | .001 | 3.54 | 3.32 | 3.08 | 2.83 | 2.67 | 2.55 | 2.41 | 2.32 | 2.25 | 2.08 | 1.92 |
| 100 | .100 | 1.66 | 1.61 | 1.56 | 1.49 | 1.45 | 1.42 | 1.38 | 1.35 | 1.34 | 1.28 | 1.22 |
| | .050 | 1.93 | 1.85 | 1.77 | 1.68 | 1.62 | 1.57 | 1.52 | 1.48 | 1.45 | 1.38 | 1.30 |
| | .010 | 2.50 | 2.37 | 2.22 | 2.07 | 1.97 | 1.89 | 1.80 | 1.74 | 1.69 | 1.57 | 1.45 |
| | .001 | 3.30 | 3.07 | 2.84 | 2.59 | 2.43 | 2.32 | 2.17 | 2.08 | 2.01 | 1.83 | 1.64 |
| 200 | .100 | 1.63 | 1.58 | 1.52 | 1.46 | 1.41 | 1.38 | 1.34 | 1.31 | 1.29 | 1.23 | 1.16 |
| | .050 | 1.88 | 1.80 | 1.72 | 1.62 | 1.56 | 1.52 | 1.46 | 1.41 | 1.39 | 1.30 | 1.21 |
| | .010 | 2.41 | 2.27 | 2.13 | 1.97 | 1.87 | 1.79 | 1.69 | 1.63 | 1.58 | 1.45 | 1.30 |
| | .001 | 3.12 | 2.90 | 2.67 | 2.42 | 2.26 | 2.18 | 2.00 | 1.90 | 1.83 | 1.64 | 1.43 |
| 1000 | .100 | 1.61 | 1.55 | 1.49 | 1.43 | 1.38 | 1.35 | 1.30 | 1.27 | 1.25 | 1.18 | 1.08 |
| | .050 | 1.84 | 1.76 | 1.68 | 1.58 | 1.52 | 1.47 | 1.41 | 1.36 | 1.33 | 1.24 | 1.11 |
| | .010 | 2.34 | 2.20 | 2.06 | 1.90 | 1.79 | 1.72 | 1.61 | 1.54 | 1.50 | 1.35 | 1.16 |
| | .001 | 2.99 | 2.77 | 2.54 | 2.30 | 2.14 | 2.02 | 1.87 | 1.77 | 1.69 | 1.49 | 1.22 |