[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 239 **G** **Your Roll No................**

Unique Paper Code : 235551

Name of the Paper : Analysis

Name of the Course : **B.A. Programme – Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. There are **3** sections.
3. Each section consists of **25** marks.
4. Attempt any **two** parts from each question in each section.
5. Marks are indicated against each question.

SECTION I

1. (a) Let A and B be non empty subsets of R and let

$$C = \{x + y : x \in A, y \in B\}$$

If each of the sets A and B has a supremum, show that C has a supremum and Sup C = Sup A + Sup B. (6)

(b) Give example of a set $S \subseteq R$ which has

(i) Exactly one limit point

(ii) Infinite number of limit points

(iii) Two limit points (6)

P.T.O.

(c) Show that function defined as $f(x) = \begin{cases} x & \text{if } x \text{ is irrational} \\ -x, & \text{if } x \text{ is rational} \end{cases}$

is continuous only at $x = 0$. (6)

2. (a) Define an open set $S \subseteq R$. Show that intersection of two open sets is again an open set but the intersection of infinite number of open sets need not be open. Justify your answer by an example (6.5)

(b) Show that the function $f(x) = \lim_{n\to\infty} \frac{x^n}{1+x^n}$ for all $x \geq 0$,

is continuous at all points except at $x = 1$. (6.5)

(c) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on [−2, 2]. (6.5)

SECTION II

3. (a) Show that $\lim_{n\to\infty} (n)^{1/n} = 1$. (6.5)

(b) If $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences such that

$\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$, then show that

$$\lim_{n\to\infty} (a_n b_n) = \left(\lim_{n\to\infty} a_n\right)\left(\lim_{n\to\infty} b_n\right) = ab.$$ (6.5)

(c) If $\lim_{n\to\infty} a_n = l$, then show that $\lim_{n\to\infty} \frac{a_1 + a_2 + \dots\dots + a_n}{n} = l$. (6.5)

4. (a) State and prove D'Alembert's Ratio Test for a positive terms series $\sum_{1}^{\infty} u_n$. (6)

(b) Test the convergence of the following series :

(i) $\frac{x}{1.3} + \frac{x^2}{2.4} + \frac{x^3}{3.5} + \frac{x^4}{4.6} + \ldots\ldots$ for all values of x.

(ii) $\sum_{1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

(iii) $\sum_{1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$ (6)

(c) Define absolute convergence and conditional convergence of an infinite series. Show that every absolutely convergent series is convergent. With the help of an example, show that the converse need. (6)

SECTION III

5. (a) Define Riemann integrability of a bounded function over [a, b]. Show that every monotonic function on [a, b] is integrable on [a, b]. (6)

(b) Discuss the convergence of the improper integral $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$. (6)

(c) (i) Define Beta and Gamma functions. What is the relation between Beta and Gamma functions ?

(ii) Show that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$ (6)

6. (a) Find the Fourier series in $[-\pi, \pi]$ for the function :

$$f(x) = \begin{cases} x, & \text{if } -\pi < x \leq 0 \\ 2x, & \text{if } 0 < x \leq \pi \end{cases}$$ (6.5)

(b) State Weierstrass's M-test for the series of the functions f_n defined on [a, b]. Show that the series $\sum_1^\infty \frac{\sin nx}{n^p}$ is uniformly convergent for all real values of x if $p > 1$. (6.5)

(c) (i) Find the radius of convergence of the power series

$$\sum_0^\infty \frac{(n+1)}{(n+2)(n+3)} x^n .$$

(ii) Discuss the Riemann integrability of the function f on [0, 3] where $f(x) = [x]$, $[x]$ is the greatest integer $\leq x$. (6.5)